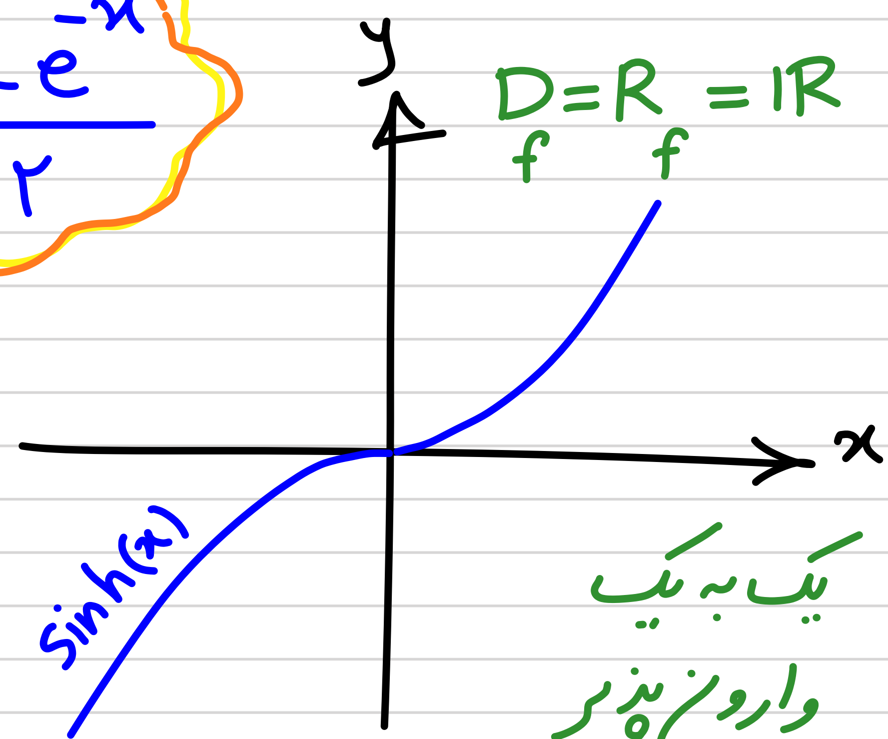
مهیار اسماعیل پور

① تابع سینوس هایپربولیک

توابع Hyperbolic

1

$$\boxed{f(x)=\sinh(x)=\frac{e^{x}-e^{-x}}{2}}$$



$D_f = R_f = \mathbb{R}$

یک به یک
وارون پذیر

$$y=\frac{e^{x}-e^{-x}}{2}$$

$$2y=e^{x}-\frac{1}{e^{x}}$$

$$2y=\frac{(e^{x})^{2}-1}{e^{x}}$$

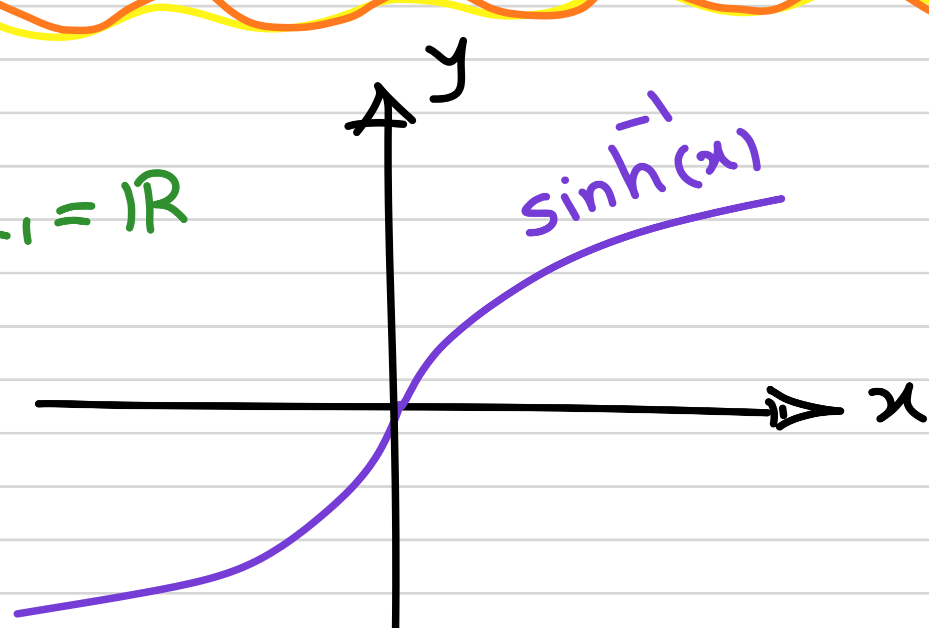
$$(e^{x})^{2}-2ye^{x}-1=0 \Rightarrow \Delta=4y^{2}+4$$

$$\begin{cases} e^{x}=\dfrac{2y-\sqrt{4y^{2}+4}}{2}<0 & \text{غ ق ق} \\ e^{x}=\dfrac{2y+\sqrt{4y^{2}+4}}{2}=y+\sqrt{y^{2}+1} \end{cases}$$

$$\Rightarrow x=\operatorname{Ln}\left(y+\sqrt{y^{2}+1}\right)$$

$$\boxed{f^{-1}(x)=\operatorname{Ln}\left(x+\sqrt{x^{2}+1}\right)=\sinh^{-1}(x)}$$

$D_{f^{-1}} = R_{f^{-1}} = \mathbb{R}$



$$(\sinh(x))' = \cosh(x)$$

$$(\sinh^{-1}(x))' = \left(\ln\left(x+\sqrt{x^2+1}\right)\right)'$$

$$= \frac{1+\dfrac{2x}{2\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{\dfrac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$$

$$(\sinh(u))' = u'\cosh(u)$$

$$(\sinh^{-1}(u))' = \frac{u'}{\sqrt{u^2+1}}$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)+c$$

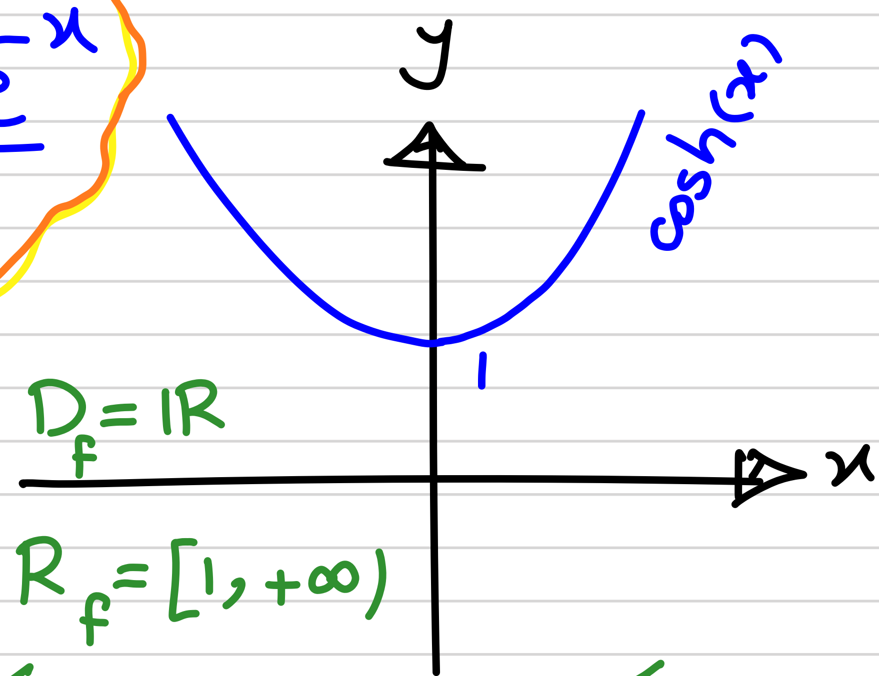
$$= \ln\left(x+\sqrt{x^2+a^2}\right)+c$$

Mahyar Esmaeilpour

@mathyar

۲) تابع کسینوس هایپربولیک

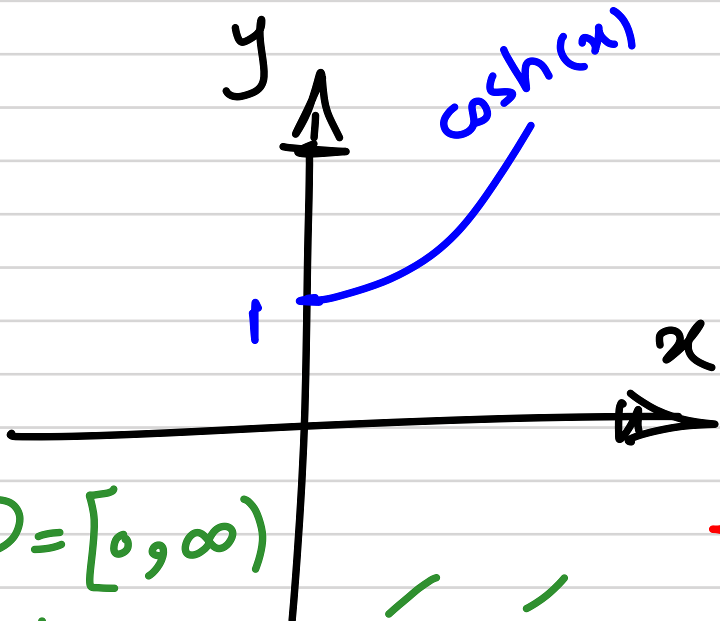
$$\boxed{f(x)=\cosh(x)=\frac{e^{x}+e^{-x}}{2}}$$



$D_f=\mathbb{R}$

$R_f=[1,+\infty)$

با محدود کردن دامنه به تابع یک به یک و وارون پذیر تبدیل می شود :



$D=[0,\infty)$ یک به یک و وارون پذیر

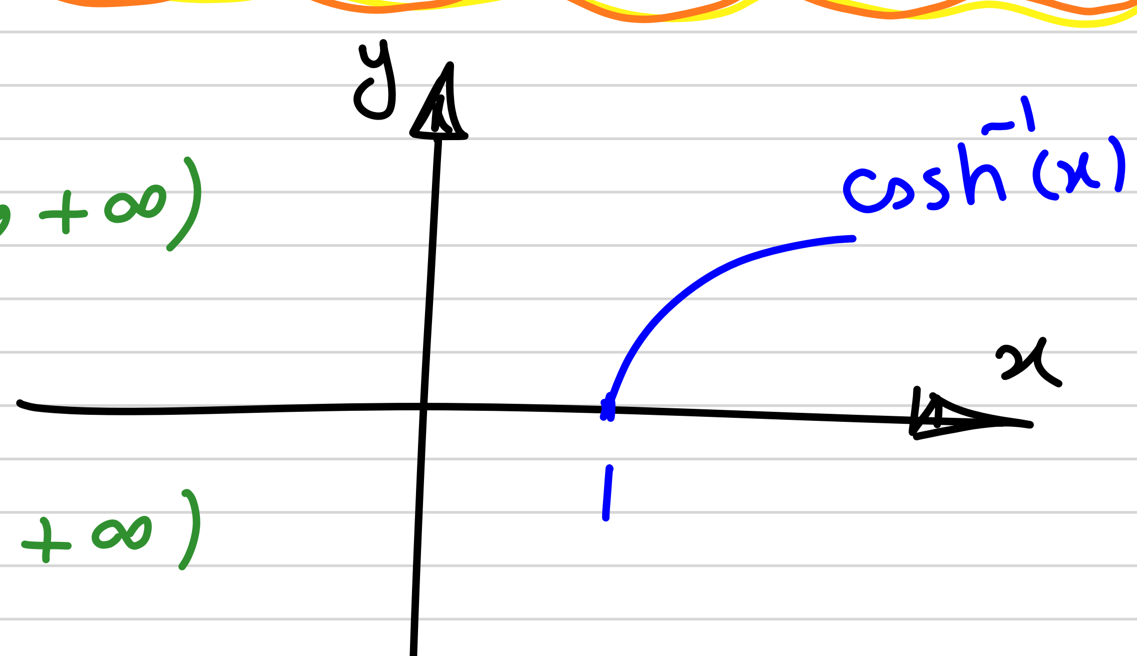
$$y=\frac{e^{x}+e^{-x}}{2} \rightarrow 2y=e^{x}+\frac{1}{e^{x}} \rightarrow 2y=\frac{(e^{x})^{2}+1}{e^{x}}$$

$$(e^{x})^{2}-2ye^{x}+1=0 \Rightarrow \Delta=4y^{2}-4$$

$$\begin{cases} e^{x}=\dfrac{2y-\sqrt{4y^{2}-4}}{2} & \text{غ ق ق} \\ e^{x}=y+\sqrt{y^{2}-1} \rightarrow x=\ln\left(y+\sqrt{y^{2}-1}\right) \end{cases}$$

$$\boxed{f^{-1}(x)=\ln\left(x+\sqrt{x^{2}-1}\right)=\cosh^{-1}(x)}$$

$D_{f^{-1}}=[1,+\infty)$



$R_{f^{-1}}=[0,+\infty)$

$$(\cosh(x))' = \sinh(x)$$

$$(\cosh^{-1}(x))' = \left(\ln\left(x+\sqrt{x^2-1}\right)\right)' = \frac{1}{\sqrt{x^2-1}}$$

$$(\cosh(u))' = u'\sinh(u)$$

$$(\cosh^{-1}(u))' = \frac{u'}{\sqrt{u^2-1}}$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)+c = \ln\left(x+\sqrt{x^2-a^2}\right)+c$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2} \quad \text{و} \quad \cosh^2 x = \frac{\cosh 2x + 1}{2}$$

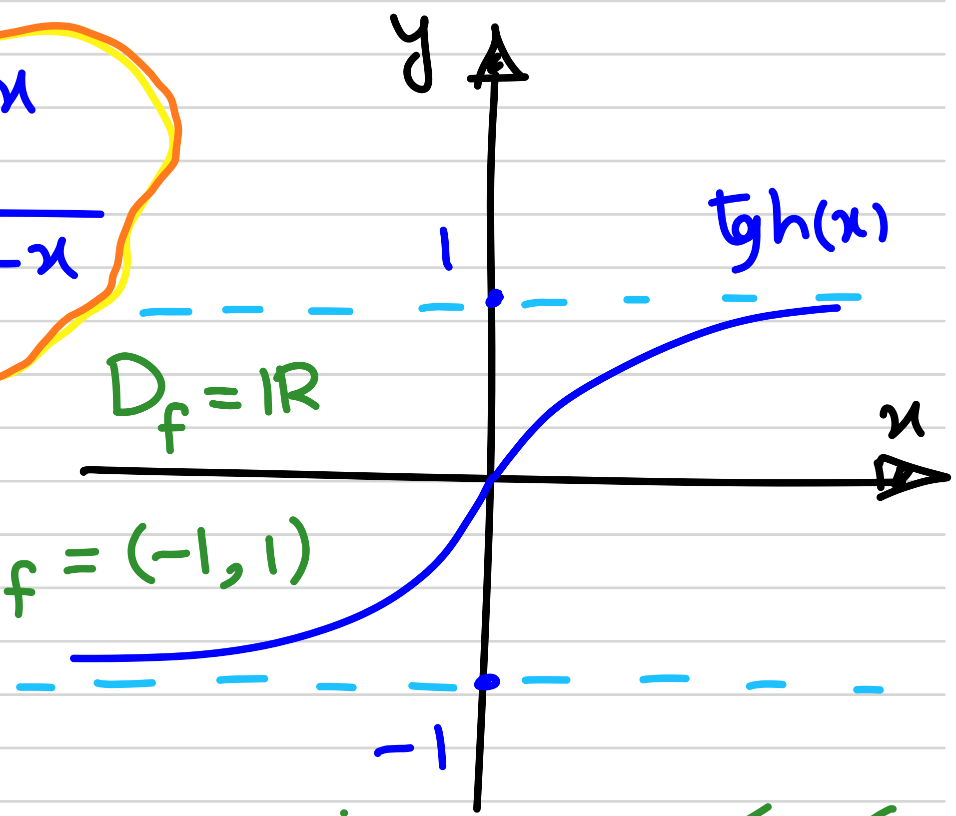
$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \begin{cases} \cosh^2 x + \sinh^2 x \\ 2\cosh^2 x - 1 \\ 1 + 2\sinh^2 x \end{cases}$$

۳) تابع تانژانت هایپربولیک

3

$$f(x)=tgh(x)=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$



$D_f=\mathbb{R}$

$R_f=(-1,1)$

یک به یک و وارون پذیر

$$y=\frac{e^{x}-\frac{1}{e^{x}}}{e^{x}+\frac{1}{e^{x}}}=\frac{e^{2x}-1}{e^{2x}+1}$$

$$e^{2x}-1=ye^{2x}+y$$

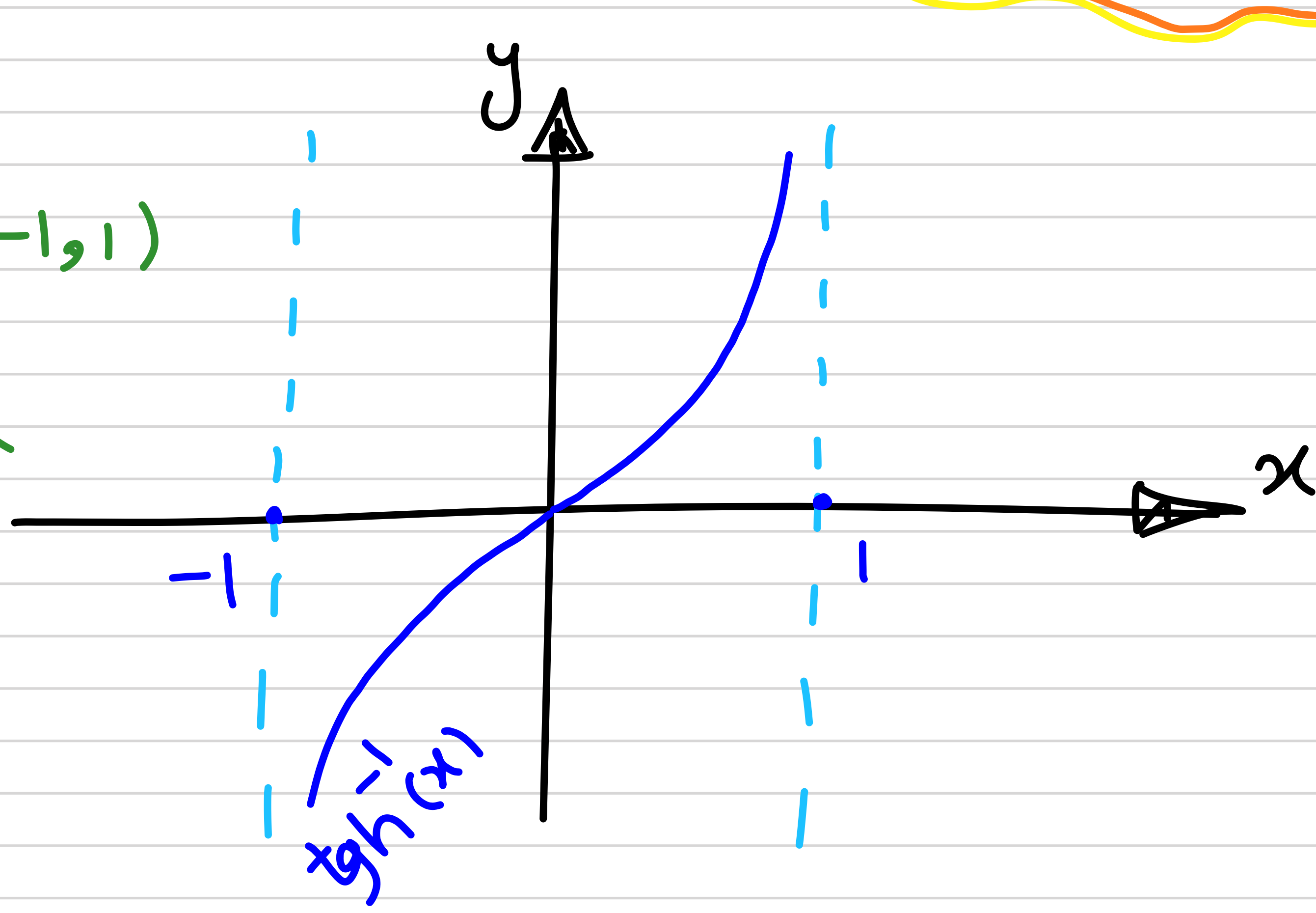
$$(1-y)e^{2x}=y+1$$

$$e^{2x}=\frac{1+y}{1-y}\xrightarrow{\text{جذر}}e^{x}=\pm\sqrt{\frac{1+y}{1-y}}\Rightarrow e^{x}=\sqrt{\frac{1+y}{1-y}}$$

$$x=Ln\sqrt{\frac{1+y}{1-y}}\Rightarrow \boxed{f^{-1}(x)=Ln\sqrt{\frac{1+x}{1-x}}}=tgh^{-1}x$$

$D_{f^{-1}}=(-1,1)$

$R_{f^{-1}}=\mathbb{R}$



@mathyar

$$(\operatorname{tgh}(x))' = \operatorname{sech}^2(x)$$

$$(\operatorname{tgh}^{-1}(x))' = \left(\operatorname{Ln}\sqrt{\frac{1+x}{1-x}}\right)' = \frac{1}{1-x^2}$$

$$(\operatorname{tgh}(u))' = u'\operatorname{sech}^2(u)$$

$$(\operatorname{tgh}^{-1}(u))' = \frac{u'}{1-u^2}$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\operatorname{Ln}\left|\frac{a+x}{a-x}\right| + c$$

$$= \frac{1}{a}\operatorname{tgh}^{-1}\left(\frac{x}{a}\right) + c$$

@mathyar

$$1-\operatorname{tgh}^2 x = \operatorname{sech}^2 x$$

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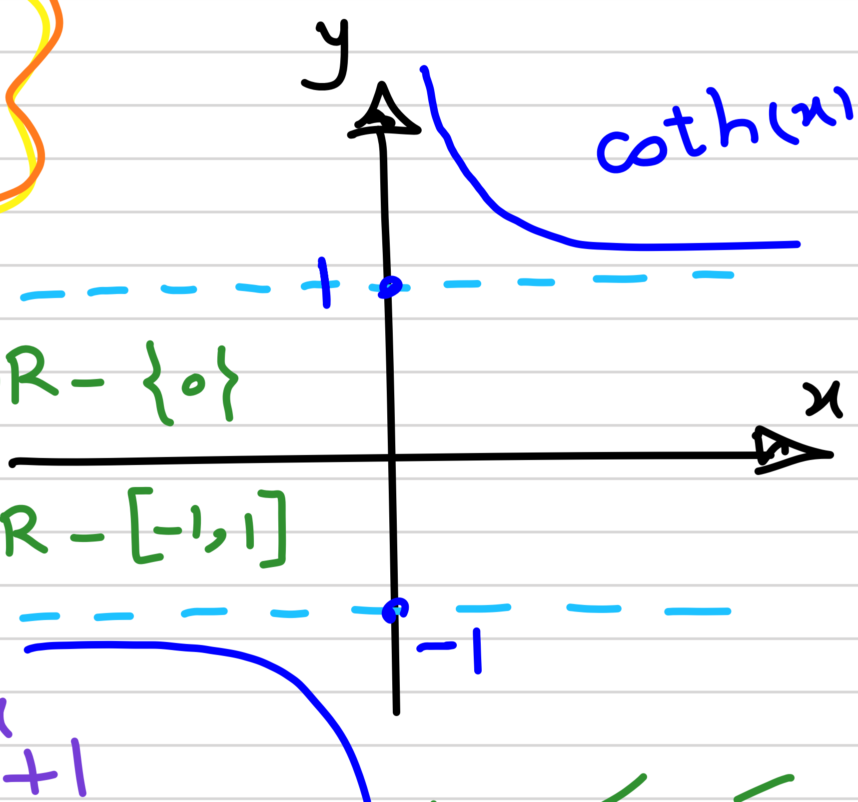
④ تابع کتانژانت هایپربولیک

4

$$\boxed{f(x)=\coth(x)=\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}}$$



$$y=\frac{e^{x}+\frac{1}{e^{x}}}{e^{x}-\frac{1}{e^{x}}}$$

$D_f=\mathbb{R}-\{0\}$

$R_f=\mathbb{R}-[-1,1]$

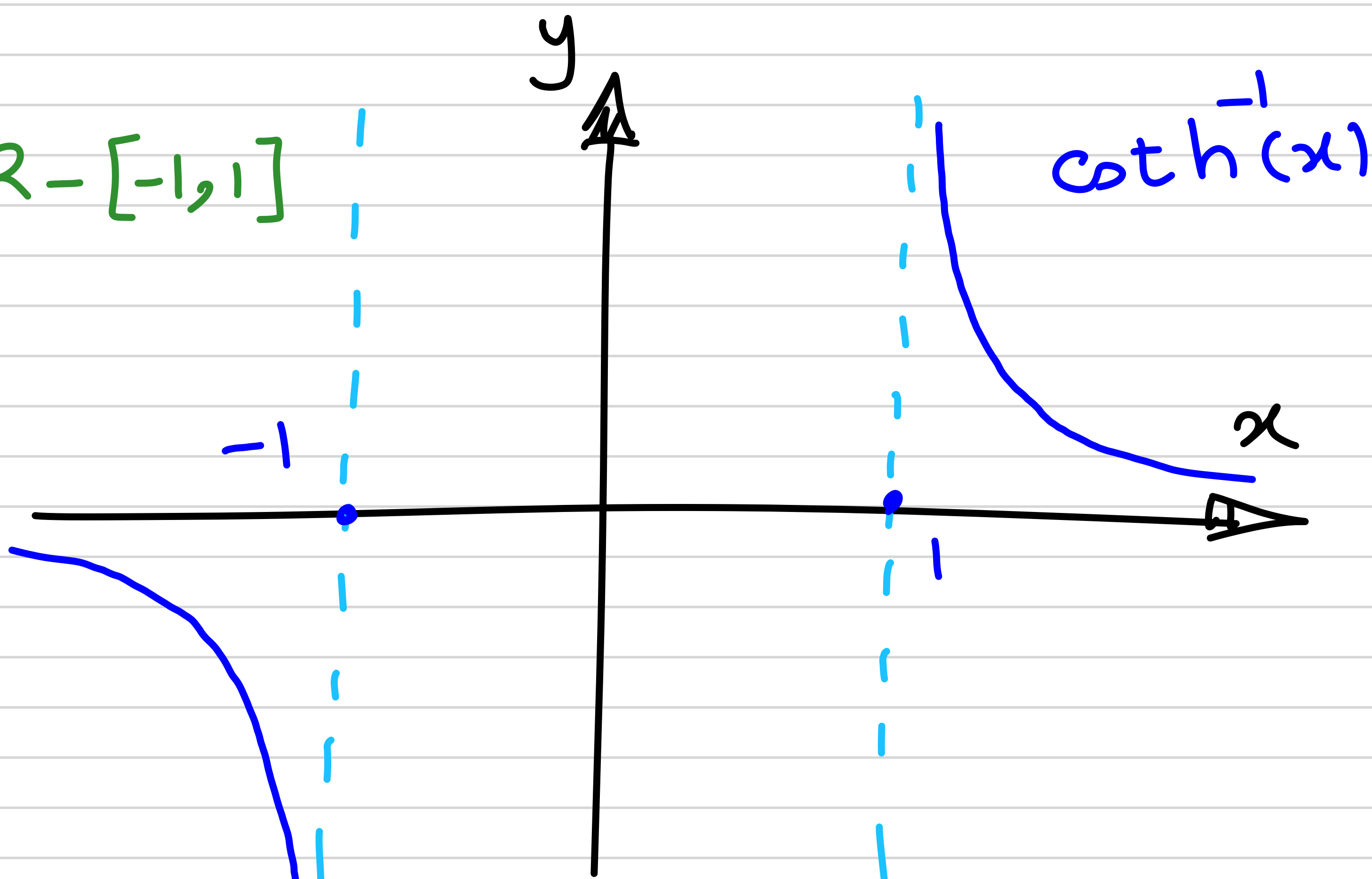
$$y=\frac{e^{2x}+1}{e^{2x}-1}\rightsquigarrow ye^{2x}-y=e^{2x}+1$$

یک به یک و وارون پذیر

$$(y-1)e^{2x}=y+1\rightsquigarrow e^{2x}=\frac{y+1}{y-1}$$

$$e^{x}=\sqrt{\frac{y+1}{y-1}}\rightsquigarrow \boxed{f^{-1}(x)=\ln\sqrt{\frac{x+1}{x-1}}}=\coth^{-1}(x)$$

$D_{f^{-1}}=\mathbb{R}-[-1,1]$



$R_{f^{-1}}=\mathbb{R}-\{0\}$

@mathyar

$$(\coth(x))' = -\operatorname{csch}^2(x)$$

$$(\coth^{-1}(x))' = \left(\ln\sqrt{\frac{x+1}{x-1}}\right)' = (\operatorname{tgh}^{-1}(x)) = \frac{1}{1-x^2}$$

$$(\coth(u))' = -u'\operatorname{csch}^2(u)$$

$$(\coth^{-1}(u))' = (\operatorname{tgh}^{-1}(u)) = \frac{u'}{1-u^2}$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$$

$$= \frac{-1}{a}\coth^{-1}\left(\frac{x}{a}\right) + c$$

@mathyar

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

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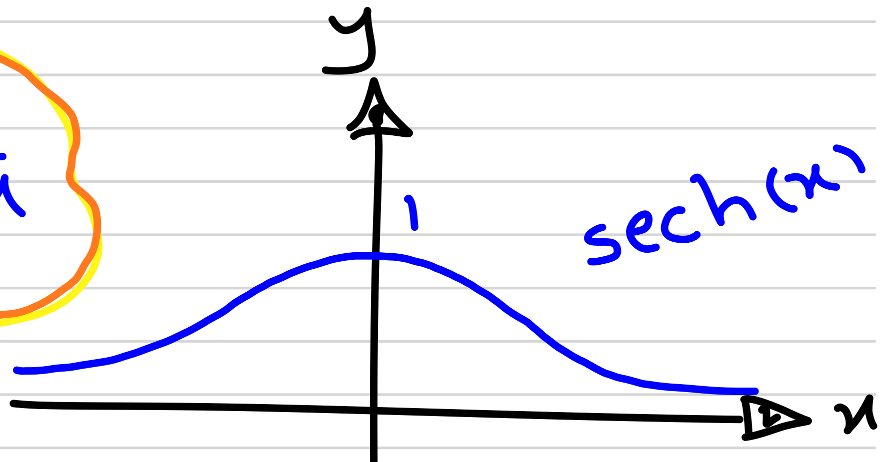
۵ تابع سکانت هایپربولیک

5

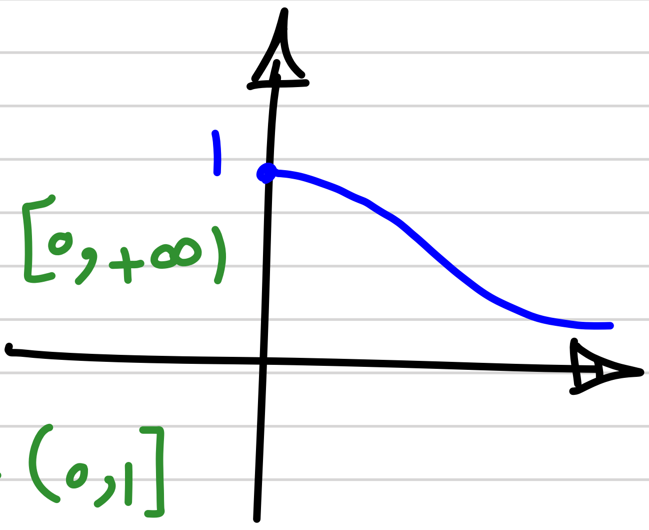
$$f(x)=\operatorname{sech}(x)=\frac{2}{e^{x}+e^{-x}}$$



$D_f=\mathbb{R}$

$R_f=(0,1]$

$D=[0,+\infty)$



$R=(0,1]$

با محدود کردن دامنه به تابع یک به یک و وارون پذیر تبدیل می شود:

$$y=\frac{2}{e^{x}+\frac{1}{e^{x}}}\Rightarrow y=\frac{2e^{x}}{e^{2x}+1}\Rightarrow ye^{2x}+y=2e^{x}$$

$$ye^{2x}-2e^{x}+y=0\Rightarrow \Delta=4-4y^{2}$$

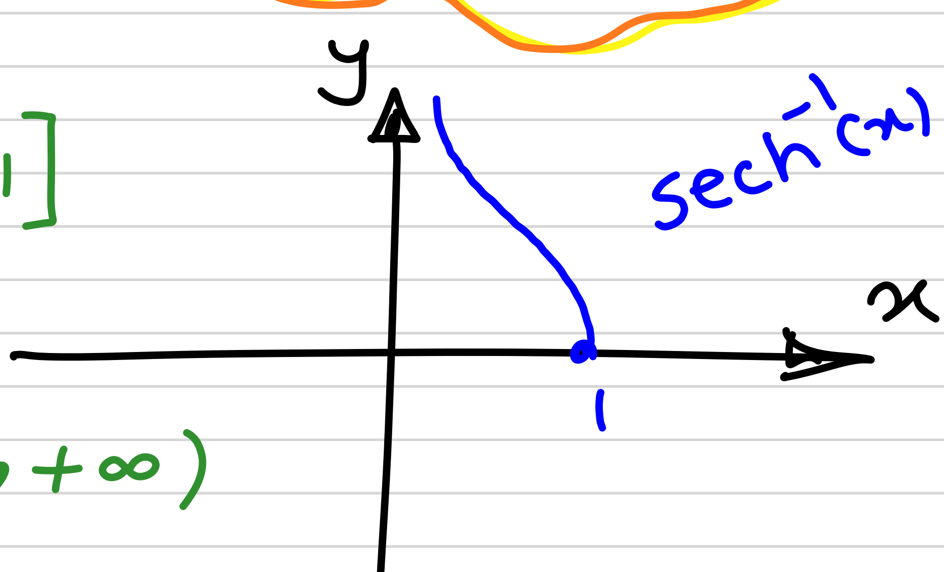
@mathyar

$$\begin{cases} e^{x}=\dfrac{1-\sqrt{1-y^{2}}}{y} & \text{غ ق ق} \\ e^{x}=\dfrac{1+\sqrt{1-y^{2}}}{y} \end{cases}\Rightarrow x=\ln\left(\frac{1}{y}+\frac{1}{y}\sqrt{1-y^{2}}\right)$$

$$f^{-1}(x)=\ln\left(\frac{1}{x}+\frac{1}{x}\sqrt{1-x^{2}}\right)=\ln\left(\frac{1}{x}\left(1+\sqrt{1-x^{2}}\right)\right)$$

$$\Rightarrow f^{-1}(x)=\ln\frac{1+\sqrt{1-x^{2}}}{|x|}=\operatorname{sech}^{-1}(x)$$

$D_{f^{-1}}=(0,1]$



$R_{f^{-1}}=[0,+\infty)$

$$\left(\operatorname{sech}(x)\right)' = -\operatorname{sech}(x)\operatorname{tgh}(x)$$

$$\left(\operatorname{sech}^{-1}(x)\right)' = \left(\operatorname{Ln}\frac{1+\sqrt{1-x^2}}{|x|}\right)' = \frac{1}{|x|\sqrt{1-x^2}}$$

$$\left(\operatorname{sech}(u)\right)' = -u'\operatorname{sech}(u)\operatorname{tgh}(u)$$

$$\left(\operatorname{sech}^{-1}(u)\right)' = \frac{u'}{|u|\sqrt{1-u^2}}$$

@mathyar

$$\operatorname{sech}^2(x) = 1-\operatorname{tgh}^2(x)$$

$$\int \operatorname{sech}(x)\,dx = \operatorname{tg}^{-1}(\sinh(x)) + c$$

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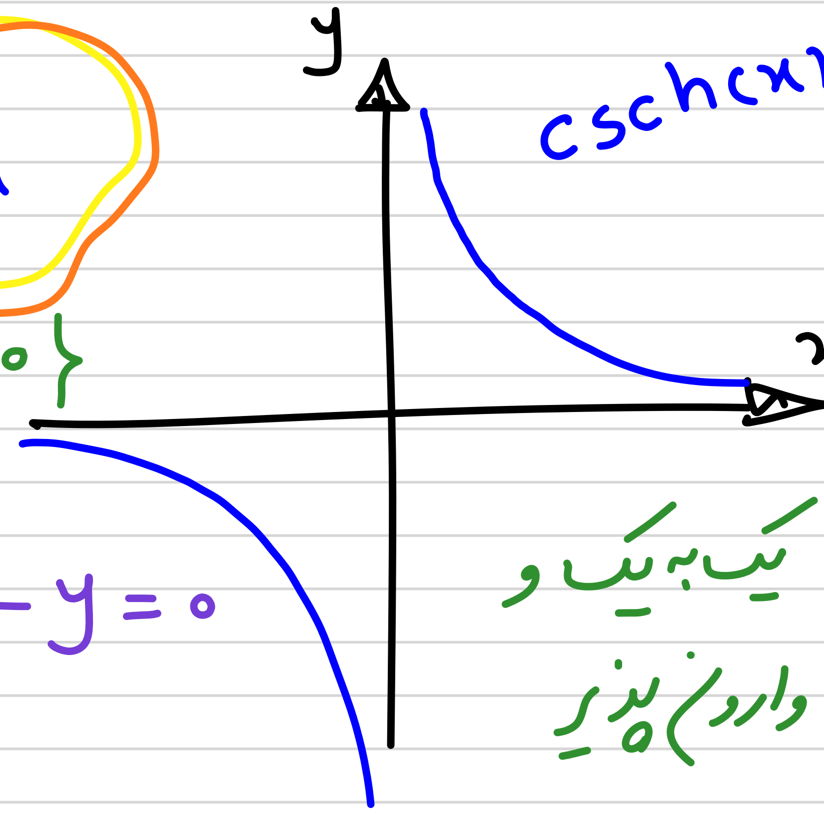
۶ تابع کسکانت هایپربولیک

6

$$f(x) = \operatorname{csch}(x) = \frac{2}{e^{x} - e^{-x}}$$

$$D_f = R_f = \mathbb{R} - \{0\}$$



یک به یک و وارون پذیر

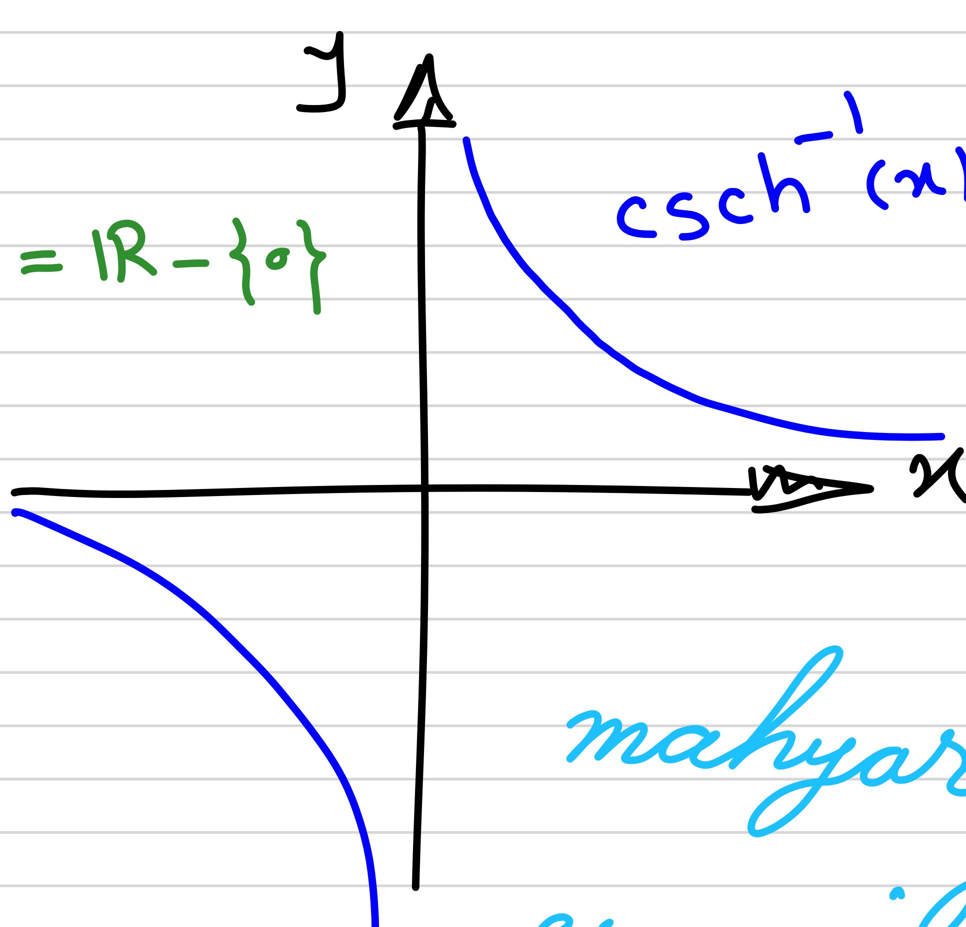
$$y = \frac{2e^{x}}{e^{2x} - 1} \rightsquigarrow ye^{2x} - 2e^{x} - y = 0$$

$$\Delta = 4 + 4y^{2}$$

$$\begin{cases} e^{x} = \dfrac{2 - \sqrt{4 + 4y^{2}}}{2y} & \text{غ ق ق} \\ e^{x} = \dfrac{1 + \sqrt{1 + y^{2}}}{y} \rightsquigarrow x = \operatorname{Ln} \dfrac{1 + \sqrt{1 + y^{2}}}{|y|} \end{cases}$$

$$f^{-1}(x) = \operatorname{Ln} \frac{1 + \sqrt{1 + x^{2}}}{|x|} = \operatorname{csch}^{-1}(x)$$

$$D_{f^{-1}} = R_{f^{-1}} = \mathbb{R} - \{0\}$$



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$$(\operatorname{csch}(x))' = -\operatorname{csch}(x)\coth(x)$$

$$(\operatorname{csch}^{-1}(x))' = \left(\frac{1+\sqrt{1+x^۲}}{|x|}\right)' = \frac{1}{|x|\sqrt{1+x^۲}}$$

$$(\operatorname{csch}(u))' = -u'\operatorname{csch}(u)\coth(u)$$

$$(\operatorname{csch}^{-1}(u))' = \frac{u'}{|u|\sqrt{1+u^۲}}$$

@mathyar

$$\operatorname{csch}^۲(x) = \coth^۲(x) - ۱$$

$$\int \operatorname{csch}(x)\,dx = \operatorname{Ln}\left|\operatorname{tgh}\left(\frac{x}{۲}\right)\right| + c$$

۱۴۰۳/۵/۲۸

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